

## Toward neural quantum computing: A theoretical and simulation study

Quantum Computing (QC) is a reality from the theoretical and practical points of view<sup>1,2,3,4,5,6</sup>. New techniques are turning quantum operations feasible<sup>1,2,3</sup>, Ion Traps (IT) among them<sup>1,2,3,4,5,6</sup>. In **IT ion traps** experiments, quantum bits (qubits) are formed from the electronic states of trapped ions and coupled through the Coulomb interaction<sup>3,4,5,6</sup>. The brain is the most sophisticated processing machine developed by nature so far, and QIC has been considered to model its function<sup>7,8,9</sup>. Dendritic spines (DS) are specialized synaptic structures (see Fig. 1) allowing large, extremely rapid, long lasting and localized  $[Ca^{2+}]$  build up at the spine<sup>10,11,12,13</sup>. They are very plastic structures<sup>14,15</sup> involved in both rapid (imprinting) and slow learning<sup>16,17,18,19</sup>. Quantum Mechanics is the most accurate description at atomic level and it lies behind all chemistry that provides the basis for biology ... maybe the magic of entanglement is also crucial for life<sup>20</sup>. In line with this reasoning, We propose DS as IT devices and Deutsch-Jozsa algorithm<sup>1,6,21</sup> (DJA) to model a Cortical Pattern Recognition Device (CPRD), and implemented a probabilistic CPRD that was successful in visual pattern recognition.

Basic to the understanding of DS as a quantum IT device is the physiology of the of the glutamate (Glu) and its receptors (Fig. 1A)<sup>22,23,24,25</sup>: **a) AMPA receptor**: the Glu-binding to AMPA allows the entry of  $Ca^{2+}$  and promotes a depolarization of the membrane potential (EM), what facilitates  $Ca^{2+}$  entry through the voltage sensitive Ca channels (VSCCs in Fig. 1); **b) NMDA receptor**: functions as a coincidence detector (CD), since the AMPA EM depolarization remove the  $Mg^{2+}$  attached to it, and a posterior Glu-binding within a temporal window  $\Delta$  (100ms), allows new  $Ca^{2+}$  to move in and to bind to calmodulin, that controls the CaMKII kinase to deliver energy to other biochemical processes; **c) mR-Glu receptors**: Glu-binding to mRGlU activates many types of **G-proteins** controlling other cellular processes. G-protein is CD,  $\tau$  ( $\Delta = 200$  to 600ms). A set of proteins<sup>22,23</sup> anchors Glu-receptors in the membrane by (Fig.1).  $Ca^{2+}$  concentration inside the cell is controlled by pumping it outside the cell or to organelles (e.g., SRE in Fig. 1A).

**Conjecture 1:** CD was proposed to create entangled quantum states in the brain<sup>7,8</sup>.

The rationality is the following (Fig. 1). **1)** A first Glu released binds to the AMPA channel promoting EM reduction, Mg<sup>2+</sup> release from the NMDA channel, VSCC activation and Ca<sup>2+</sup> entry. These Ca<sup>2+</sup>s are physically trapped by proteins in a molecular cavity of typical size of 0.7 Å in the spine. **2)** A second release of Glu within Δ=100ms, allows new Ca<sup>2+</sup> to bind to calmodulin to provide energy to create state superposition and entanglement of the trapped Ca<sup>2+</sup> ions. The electronic qubits are |0> = s<sup>1</sup> and |1> = <sup>3</sup>p<sup>0</sup> and the required energy is in the band of 50 nm (U.V.). The motion qubits are |0> = ground state and |1> = first excited state, with ground state energy of 3.2 eV. **3)** A third Glu release over mR-Glu within Δ>100ms, activates G-proteins to deliver energy to manipulate the entangled Ca<sup>2+</sup> and to move it into the SRE.

Deutsch-Jozsa's algorithm (DJA) was the first explicit example of a computational task performed exponentially faster using quantum effects than by classical means<sup>1,6,21</sup>.

Given a one-bit function  $f$ , one have: **a)** two constant functions  $f(0)=0$  and  $f(1)=0$ , or  $f(0)=1$  and  $f(1)=1$ ; or **b)** two "balanced"  $f(0)=0$  and  $f(1)=1$ , or  $f(0)=1$  and  $f(1)=0$ . A one qubit QC (Fig. 1B) is able to decide if  $f$  is constant or balance in just one step. Starting with the standard state |0> |0> at the input (I) and output (O) registers, a NOT operation is applied to I and the Hadamard transformation (H in fig. 1b) is applied to both registers. Thus:

$$\begin{array}{ccc}
 & \text{NOT} & \text{H} \\
 |0\rangle |0\rangle & \longrightarrow & |0\rangle |1\rangle \longrightarrow \left( \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \quad (1)
 \end{array}$$

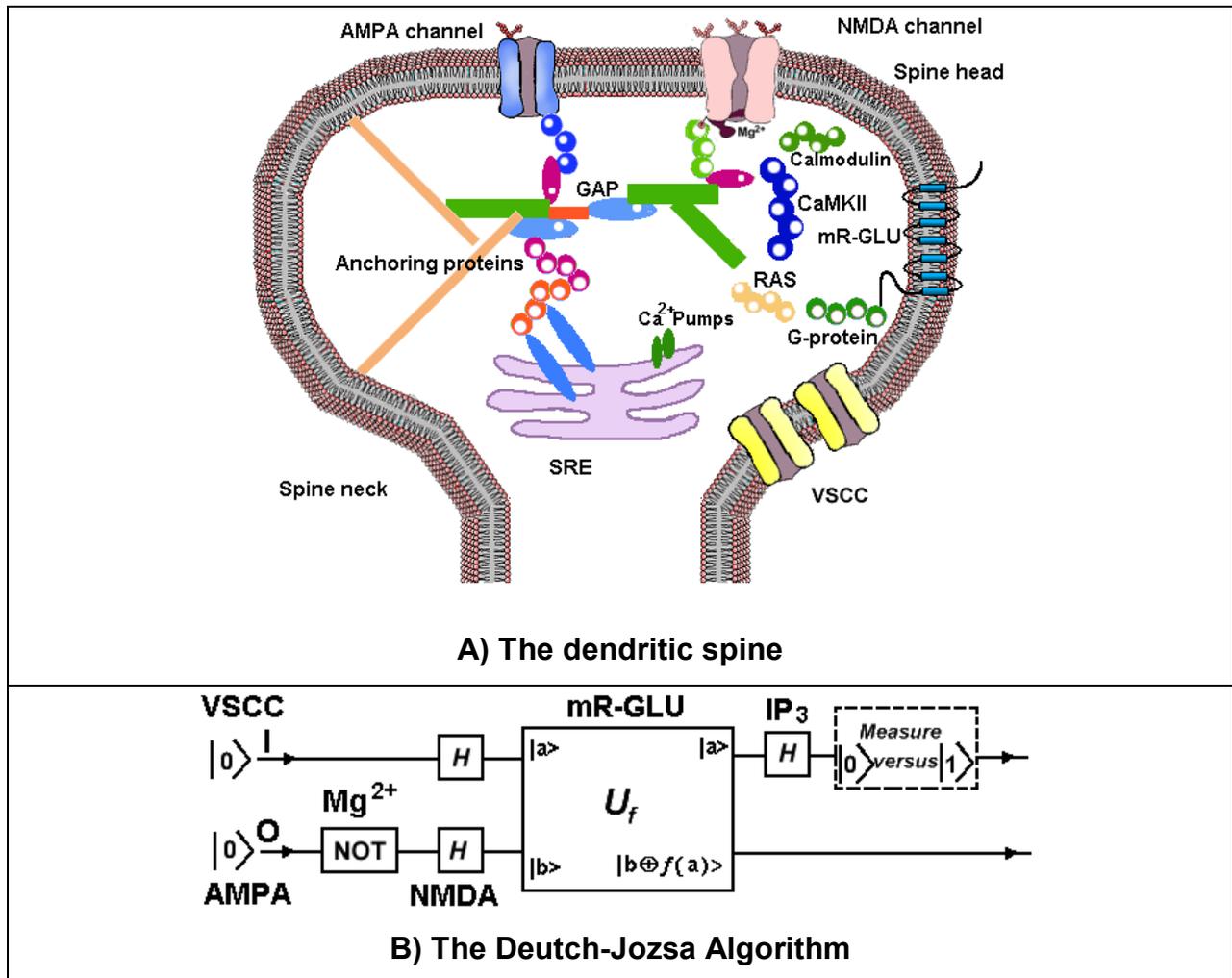
Next, the unitary transformation  $U_f$  is applied to both registers:

$$U_f: \left( \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \rightarrow \left( \frac{1}{\sqrt{2}} \sum_{x \in B} (-1)^{f(x)} |x\rangle \right) \left( \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \quad (2)$$

In this condition, O remains in the state  $(|0\rangle - |1\rangle / \sqrt{2})$  and I is left in the state  $(\ )$ .

Therefore, if  $f$  is constant I is  $\pm(|0\rangle - |1\rangle / \sqrt{2})$  and if  $f$  is "balanced" it is  $\pm(|0\rangle + |1\rangle / \sqrt{2})$ .

If  $H$  is applied again to  $I$ , it becomes  $\pm(|0\rangle)$  if  $f$  is constant and  $\pm(|1\rangle)$  if  $f$  is “balanced”. These states are reliably distinguished by a measurement in the standard basis, thus distinguishing balanced from constant functions after just one query. DJA was implemented in a  $Ca^{2+}$  ITC<sup>6</sup>.



**Fig. 1 – The dendritic spine as a Quantum Computing Device**

- A)  $Ca^{2+}$  moved into the dendritic spines through AMPA and VSCC channels and are trapped in the narrow gaps between the proteins of the post-synaptic density by the electric field generated the membrane depolarization.  $Ca^{2+}$  entering the NMDA channel binds to Calmodulin and this  $Ca^{2+}$  bounded protein (CaMKII) activates RAS, which under the action of GAP liberates energy to drive the  $Ca^{2+}$  logic gates. G-proteins are also used for the same purpose, because they also activates RAS. Finally, the trapped  $Ca^{2+}$  is moved into the Smooth Endoplasmic Reticulum (SER) by cAMP activated  $Ca^{2+}$  pumps.
- B) The schematic DJA quantum computation and its correlation with the glutamate and voltage controlled  $Ca^{2+}$  channels. See text for further details.

**Conjecture 2:** It is supposed here, that DS process the DJA as follows (Fig. 1). The AMPA channel is associated to **O** whereas the VSSC is assigned to **I**. EM depolarization promoted by  $\text{Ca}^{2+}$  entry through the AMPA channel remove the  $\text{Mg}^{2+}$  from the NMDA channel and opens VSSCs. Next, NMDA channel are activated, and CaMKII is used to perform the Hadamard transformation upon the  $\text{Ca}^{2+}$  trapped in the spine head. Next, mR-Glu receptor is activated, and a G-protein is used to implement  $\mathcal{U}_f$ . The same or another G-protein releases energy to perform another Hadamard transformation. Finally the  $\text{Ca}^{2+}$  transporting into SER is used to read the result. A point worth to remark is that DS is amenable to many different and sophisticated experimental manipulations, see for instance 10,<sup>11,12,13,14,15,16</sup>, that may be used to check these assumptions.

Now, let's suppose another function  $f : \mathbf{B}^n \rightarrow \mathbf{B}$  that is either constant if the  $2^n$  values are either 1 or 0, or balanced if exactly half (i.e.  $2^{n-1}$ ) of the values are 0 and half are 1.  $\text{DJA}^{1,21}$  is extended to treat such kind of function, if one starts with a row of  $n$  **I** qubits and one **O** qubit and to applies the same step procedures above. At the end, the  $n$  **I**s are in state

$$|\xi_f\rangle = \left( \frac{1}{\sqrt{2^n}} \sum_{x \in 2^n} (-1)^{f(x)} |x\rangle \right) \quad (3)$$

If  $f$  is constant then  $\xi_f$  will be just an equal superposition for all the  $|x\rangle$ 's with an overall plus or minus sign, whereas if  $f$  is a balanced function then  $\xi_f$  will be an equally weighted superposition with exactly half of the  $|x\rangle$ 's having the minus signs. Recalling that **H** has its own inverse ( $\mathbf{H}\mathbf{H}=\mathbf{1}$ ) and that **H** applied to each qubit  $|0\rangle$  of  $|\xi_f\rangle$  results in an equal superposition of all  $|x\rangle$ 's. Therefore, if  $f$  is constant then the resulting state is  $x=\pm|0\rangle|0\rangle\dots|0\rangle$ , and if it is balanced then  $|x\rangle$ 's is  $x\neq|0\rangle|0\rangle\dots|0\rangle$ . The reading of each of the  $n$  qubits completes the measurement. DJA requires  $O(n)$  steps to distinguish balanced from constant functions, whereas classical algorithm demand  $O(2^n)$  steps for the same task. However, a probabilistic algorithm is able to solve the same task in  $k$  steps with a probability of  $(1-\zeta)$  for correct answer and  $\zeta$  less  $1/2^k$ .

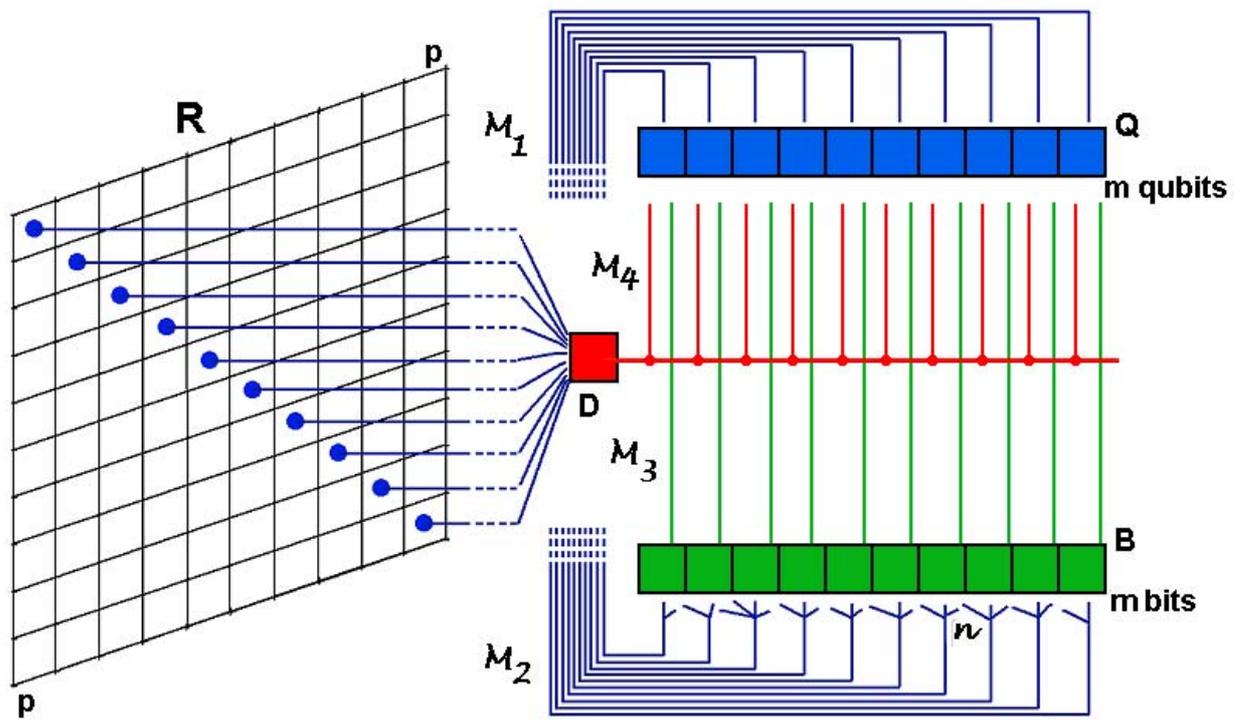
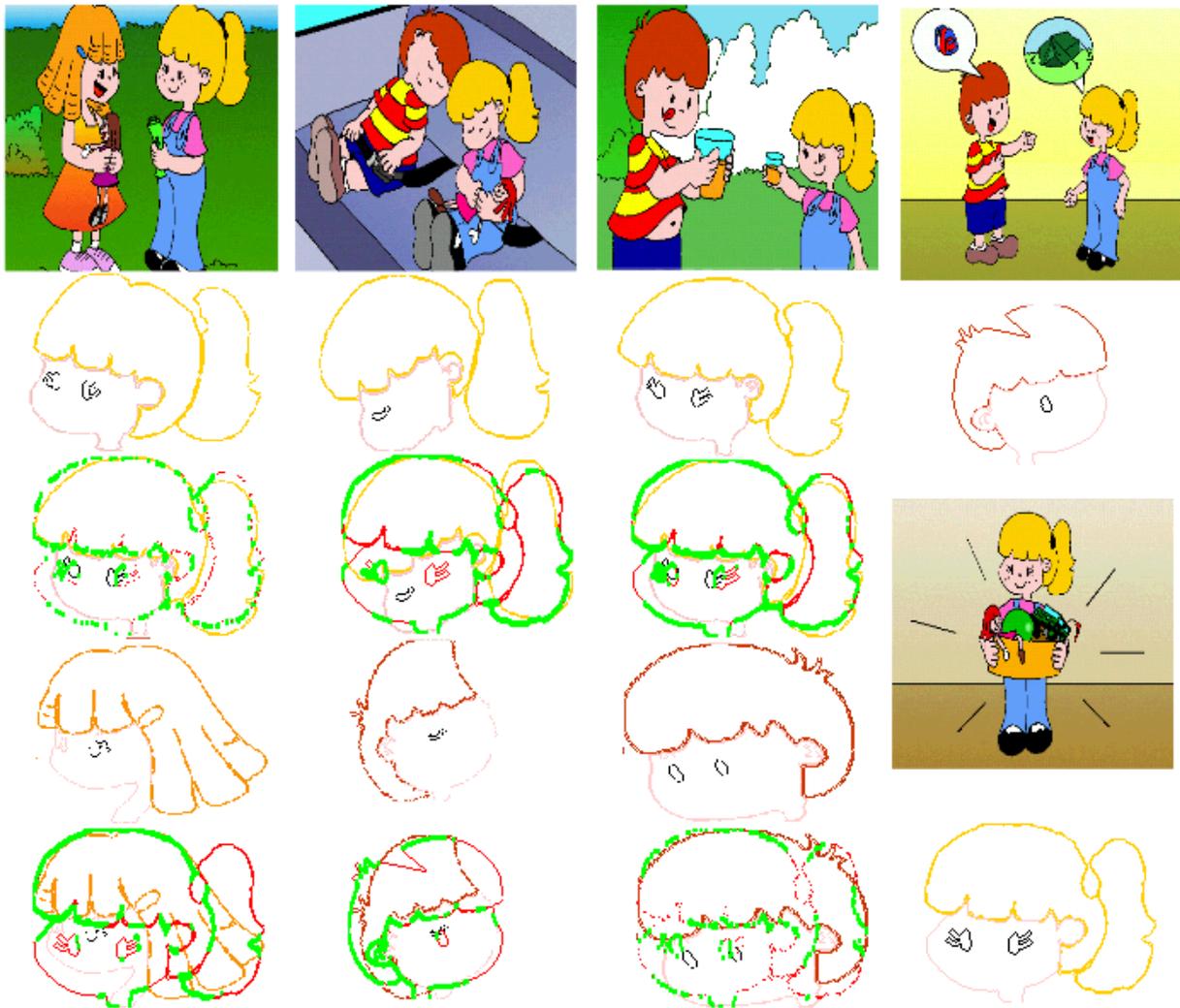


Fig. 2 – A model of QCPRD

Let the DJA processing device **CPRD** in fig. 2 be formed by the:

- a) retina **R** as an array of  $r=pxp$  bits;
- b) device **B** (cortical stelate cells) of  $m$  states of 1 bit;
- c) device **Q** (cortical pyramidal cells) of  $m$  states of 1qubit;
- d) **AND** device **D** (cortical stelate cell) of 1 qubit;
- e) measuring device **M** of 1 bit;
- f)  $M_1$  mapping  $n_1$  ( $n_1=1$  in fig.2) neighbor bits in **R** to a bit of **Q**;
- g)  $M_2$  mapping  $n_2$  neighbor bits in **R** to a bit of **B**;
- h) one to one mapping  $M_3$  from **B** to **Q**;
- i) one to all mapping  $M_4$  from **D** to **Q**,
- j) one to all mapping  $M_4$  from **Q** to **A**;
- k) a function  $f(x)$ , such that either  $f(0) = f(1)$  and it is said constant or  $f(0) \neq f(1)$  and  $f(x)$  is said to be balanced.



**Fig. 3 – Examples of PRCD recognition**

Learning and testing were done according to the following steps:

- 1) select a searching element: the element to be used to select the figures in the data base. In the case of the example of Fig. 7, the chosen element was the girl Laura® and the boy Juca;
- 2) select the element features to be used in the search: these are the components of the query to be used to search data base figures. In the present example, the query features were the face, eyes and hair contours;
- 3) use the query elements for PCPRD training: one example is used in the case of imprinting simulation, and more than one training figure in the case of slow learning.

test the efficacy of the system using another set of figures: a new set of figures containing and not containing the query element, is used to study PRCD efficacy in correctly identifying the figures containing the query element, without missing any one or mistakenly taking other elements as the query one. See [www.eina.com.br/sensor](http://www.eina.com.br/sensor) for details about the above processing.

This CPRD structure is modeled upon the visual cortical column<sup>26,27,28</sup>, specially concerning the behavior of the device **D** and it relations with  $Q^{26}$ .

**Algorithm 1:** CPRD is assumed here a QCPRD if:

- 1) the **D** bit is set 1 at the moment  $t_1$  if all  $m$  bits of **R** mapping to it are set 1 at time  $t_0$  ( $t_0 < t_1$ ) otherwise it is set 0;
- 2) the input register **I** (or VSCC channel) of **Q** is set by **R** and the output register (or AMPA channel) is set by **D** at time  $t_1$ ;
- 3) If  $0 < t_1 - t_0 < \alpha$  then  $m$  bits of **Q** are set in superposition by Hadamard gates **H** triggered by the NMDA channel, and
- 4) a bit of **B** is set to 1 at the moment  $t_1$  if the  $n$  neighbor bits of **R** mapping to it are set 1, otherwise it is set 0;
- 5) the function  $f(x)$  at the  $i^{\text{th}}$  bit of **R** is set at time ( $t_1 < t_2$ ) by the mR-Glu system as constant if the  $i^{\text{th}}$  bit of **B** is 1, otherwise the function is set as balanced, and the  $m$  qubits of **R** are transformed at the time  $t_3$  by the unitary matrix

$$U_f : |x, d\rangle \rightarrow |x, d \oplus f(x)\rangle$$

- 6) after this a G-Protein applies another **H** to **I** at time  $t_4$ ,
- 7) **M** (e.g.,  $Ca^{2+}$  pumping to SER) measures **I** states of **I** in  $O(n)$  steps at time  $t_5$  by the **M**, such that its bit is set 1 if  $f(\mathbf{R})$  is constant, otherwise it is set 0, and
- 8) finally the stimulus pattern **P** at the retina **R** is recognized if  $f(\mathbf{R})$  is constant. This pattern is defined by the maps  $M_1$  and  $M_2$ , and it is a diagonal in fig.2. The complexity of **P** is directly related to the number of bits used by **B** and **Q**.

**Algorithm 2:** A probabilistic version PCPRD of CPRD is defined if **Q** is assumed to be classical device and **M** performs probabilistic measurements requiring that the majority of the readings to be 1.

**Algorithm 3:** The learning of the maps  $M_1, M_2$  is implemented by beginning with large neighborhoods  $n_1, n_2$  and pruning  $n_2$  those connections at **B** and **Q** not used for **P**

identification. If  $n_1, n_2$  are initially very large, any  $\mathbf{P}$  presented to  $\mathbf{R}$  will be associated to a constant  $f(\mathbf{R})$  and assumed as a known  $\mathbf{P}$ . Now:

- a) a rapid learning (imprinting) occurs if most (if not all) not used connections of  $M_1, M_2$  are drastically ( $n'_1, n'_2 \rightarrow 1$ ) pruned. In this condition,  $\mathbf{P}$  is fixed and easily recognized in any other future occasion.
- b) a slow learning occurs if  $n'_1, n'_2 \rightarrow \theta \ll r$  bits, the learning velocity being inversely proportional to  $\theta$ .

Imprinting is efficient if the variability  $v(P)$  of  $\mathbf{P}$  in the closed interval  $[0,1]$  is low, otherwise the number  $q$  of QCPRD specialized to recognize  $\{\mathbf{P}_i\}_{i=1 \text{ to } r}$  increases as  $r \rightarrow 1$  and  $v(P) \rightarrow 1$ . Slow learning is preferable if  $v(P)$  is large, since it allows the selection of the most significant  $s$  bits of  $\mathbf{P}$  to be used in its recognition. However, the  $s \rightarrow 0$  as  $v(P) \rightarrow 1$ . Again, different QCPRD may be created to recognize sets similar  $\{\mathbf{P}_i\}_{i=1 \text{ to } r}$ . Spine pruning is demonstrated in the case of imprinting and slow learning in the zebra finch<sup>17,19</sup>. Sexual imprinting occurs mainly at the lateral neo-hyperstriatum (LNH) and slow learning at archineostriatum caudale (ANC). Both types of learning requires an initial increase of the number of spines in both (hormonally induced) LNH and (environmentally induced) ANC. Spine density decreases in LNH within 2 days after exposition of to the female, whereas but requires around 3 weeks to stabilize in ANC. The density increase promoted by changing the animal from isolation to a social condition occurs in 3 days.

Algorithms **2** and **3** were implemented in the system **Sensor** being developed in our laboratory since 1997<sup>7</sup> ([www.eina.com.br/sensor](http://www.eina.com.br/sensor)) to simulate natural vision systems. PCPRD efficacy in learning to identified defined images was tested using the ENSCER® Data Base ([www.enscer.com.br](http://www.enscer.com.br)).

**PCPRD** attained an efficiency of 90% in cases similar to that of Laura's identification ( Fig. 3) and imprinting simulations, when dendritic spine pruning preserved neighborhoods of 10% or smaller around the query representative points and the probabilistic identification was set to admit an error of 10%. Imprinting worked fine with query elements that are very prototypical as Laura. In the case of more variable query

elements, slow learning was a better approach to characterize specific points of small of neighborhood variance, that are better key elements for a successful query. Such a characterization depends on successive pruning promoted by a given family of training examples. The **PCPRD** algorithm increased SENSOR efficiency when compared to the previous contour identification algorithm in use, based on comparison of those points at which the contour experienced a significant change of direction<sup>7</sup>.

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